An Example of Utilizing Stochastic Approach for Investigating Network Constraints (Congestions) on Horizontally Operated Power Systems with Distributed Generation

M. Reza, G. Papaefthymiou, P. H. Schavemaker

1Electrical Power System Laboratory, Delft University of Technology, the Netherlands
2TenneT TSO (Transmission System Operator) bv, the Netherlands

Abstract

This paper shows an example of a stochastic approach to study the impact of distributed generation (DG) on the network constraints (congestions) in power systems. We assume the DG units to be customer-owned, so that they can be connected to or disconnected from the power system by their owners at random. Therefore, the DG units generate power in a stochastic way. The load in the system shows a random behavior too, and the probability distribution of the aggregated generated power and load demand of the distribution system are calculated using Monte Carlo Simulation (MCS). The network constraints (congestions) are evaluated based on the probability distributions of the power flows that result from the simulations. The method that is applied in this paper, shows that looking at the network constraints with a stochastic approach gives a more complete picture of the network than applying a deterministic method, especially when non-dispatchable DG units play a dominant role.

Keywords: power system, distributed generation, power flows, network congestions, stochastic approach, Monte Carlo simulation

Introduction

Power Balance Problem in Power Systems

A power system is designed to supply electrical power to the customers. Up to now, electrical power is generated by means of large synchronous generators located in power plants that are mainly constructed at remote sites (relatively far away from the customers/load centers). In this ‘traditional’ way, a power system consists of three parts, namely: generation, transmission, and distribution. The generation system converts mechanical power (that results from the conversion of primary energy sources, such as: hydro power, coal, gas, etc.) into electrical power, the transmission system transports the electrical power over long distances to the load centres, and the distribution system distributes the electrical power to the customers/loads. Fig. 1 shows a schematic representation of this primary structure of the power system [6].

In power systems, power production must be in balance with the consumption (plus losses that occur in the network) at any time. This is due to the fact that electrical energy cannot be stored in large quantities (yet).
'Traditional' Vertically-Operated and 'Future' Horizontally-Operated Power Systems

Traditionally, the power system is operated according to the schematic displayed in fig. 1. Nowadays, a lot of (relatively) small-scale generators are connected to the distribution network, close to the load centers. This trend is driven by many factors especially the environmental one. These small generators are indicated as Distributed Generation (DG) [4], [5].

When the number of DG units in the power system increases to a large extent, the power system can transform from the conventional 'vertically-operated power system' towards a future 'horizontally-operated power system' [8]. In fig. 2, the traditional 'vertically operated power system' (upper graph) is compared with a 'horizontally operated power system' (lower graph). Note that the term 'centralized generators' in fig. 2 corresponds to the term 'Power generation' in fig. 1, the term 'transmission network' corresponds to the term 'Power transmission', and the term 'loads' or 'loads and distributed generators' correspond to the term 'Power distribution'. In the new 'horizontal' power system the distribution networks become 'active': the power is not only consumed but also generated in the distribution networks, and the direction of power flows in the network is not fixed any more [8].

Load Flow Computations of Distributed Energy Systems (DES)

The Load Flow Computation

The load flow (or power flow\(^1\)) calculation. This computation gives insight in the state of a power system for a specific steady-state situation, as it computes, amongst others, the voltage at each bus (magnitude and phase angle) and the power flow (real and reactive) in each line [2]. The load flow computation enables us to analyze the (steady-state) performance of the power system under various operating conditions.

The Load Flow Computation

The load flow computation is the determination of the voltage (magnitude and phase angle) at each bus of the power system under specified conditions of

\(^1\) Note that in this paper both the terms 'load flow' and 'power flow' are used as they refer to the same computation
operation (i.e. specified power consumption of the loads and power production at each generator). In mathematical terms, the load flow problem is nothing more than a system consisting of as many non-linear equations as there are parameters to be determined.

Solving the load flow problem starts from obtaining the data in form of the single-line diagram of power systems, where the transmission lines – represented by the equivalent circuits and the numerical values for the series impedance $Z$ and the total line-charging conductance $Y$ – are used to determine all elements of the $N \times N$ bus admittance matrix of a system (with $N$ buses), with the typical element $Y_{ij}$ of this matrix is:

$$Y_{ij} = Y_{ij} | \angle \delta_{ij}$$

$$Y_{ij} = |Y_{ij}| \cos \delta_{ij} + j|Y_{ij}| \sin \delta_{ij}$$

where $Y_{ij}$ is the admittance element between bus $i$ and $j$, which is a complex number represented in polar coordinates via magnitude $|Y_{ij}|$ and angle $\delta_{ij}$ and in cartesian ones by $G_{ij}$ and $B_{ij}$ where $G_{ij}, B_{ij}$ are the conductance and susceptance of the element $Y_{ij}$. The voltage $V$ at a typical bus $i$ of the system is given as:

$$V_i = |V_i| \angle \delta_i$$

$$V_i = |V_i| (\cos \delta_i + j \sin \delta_i)$$

and the current injected into the network at bus $i$ in terms of the elements $Y_{in}$ of $Y_{bus}$ is given as:

$$I_i = Y_{i1}V_1 + Y_{i2}V_2 + \ldots + Y_{in}V_n$$

$$I_i = \sum_{n=1}^{N} Y_{in}V_n$$

Furthermore, according to this representation, the load flow equations can be obtained as:

$$P_i = |V_i|^2 G_i + \sum_{n=1,i \neq i}^{N} |V_i| V_n Y_{in} \cos (\theta_n + \delta_n - \delta_i)$$

$$Q_i = -|V_i|^2 B_i + \sum_{n=1,i \neq i}^{N} |V_i| V_n Y_{in} \cos (\theta_n + \delta_n - \delta_i)$$

where $P_i, Q_i$ are the active and reactive power injections at node $i$, $G_i, B_i$ are the conductance and susceptance of the element $Y_{in}$ of the admittance matrix and $|V_i|$ and $\delta_i$ are the voltage magnitude and angle at node $i$. Using eq. 4 and eq. 5, the calculated values for the net real power $P_i$ and reactive power $Q_i$ entering the network at typical bus $i$ are provided. Denoting $P_{gi}$ as the scheduled power being generated at bus $i$ and $P_{di}$ as the scheduled power demand of the load at bus $i$, the $P_{sch}$ as the net scheduled power being injected in the network at bus $i$ is defined as:

$$P_{i,sch} = P_{gi} - P_{di}$$

Furthermore, denoting the calculated value of $P_i$ by $P_{calc}$ leads to the definition of mismatch $\Delta P_i$ and $\Delta Q_i$ as:

$$\Delta P_i = P_{i,sch} - P_{i,calc}$$

$$\Delta Q_i = Q_{i,sch} - Q_{i,calc}$$

Thus, mismatches occur when calculated values of $P_i$ and $Q_i$, i.e. $P_{calc}$ and $Q_{calc}$ do not coincide with the scheduled values $P_{sch}$ and $Q_{sch}$. Furthermore, if the calculated values $P_{calc}$ and $Q_{calc}$ match the scheduled values $P_{sch}$ and $Q_{sch}$ perfectly, the mismatches $\Delta P_i$ and $\Delta Q_i$ become zero and the power balance equations can be written as:

$$g_i = P_i - P_{i,sch}$$

$$g_i = Q_i - Q_{i,sch}$$

Each bus of a power network has two above equations. Thus, the power flow problem is actually to solve eq. 4 and eq. 5, for values of the unknown bus voltages that cause eq. 10 and eq. 11 to be numerically satisfied at each bus. If there is no scheduled value $P_{sch}$ for bus $i$, the mismatch $P_i = P_{calc} - P_{sch}$ cannot be defined and there is no requirement to satisfy the corresponding eq.10 in the course of solving the power-flow problem. Similarly, if there is no scheduled value $Q_{sch}$ for bus $i$, the mismatch $Q_i = Q_{calc} - Q_{sch}$ cannot be defined and there is no requirement to satisfy the corresponding eq. 11 in the course of solving the power-flow problem. Since each bus $i$ in a power system may be associated with four potentially unknown quantities, namely: $P_i$, $Q_i$, voltage angle $\delta_i$, and voltage magnitude $|V_i|$, and with at most, that there are two equations like eq. 10 and eq. 11 available for each node. Thus, it is practical in power flow study, to identify three types of buses in the network where at each bus $i$, two of the four quantities $P_i$, $Q_i$, $\delta_i$, and $|V_i|$ are specified and the remaining two are calculated. These three types of buses are:

1. **Load buses**, where the quantities of $P_i$ and $Q_i$ are specified.
2. **Voltage-controlled buses**, where the quantities of $P_i$ and $|V_i|$ are specified.
3. **Slack bus**, where the quantities of \( \delta \) and \( |V| \) are specified.

Note that the unscheduled bus-voltage magnitudes and angles in the input data of the power-flow study are called state variables or dependent variables since their values, which describe the state of the systems, depend on the quantities specified at all buses. Hence, the power-flow problem is to determine values for all state variables by solving an equal number of power-flow equations based on the input data specifications, and because the functions \( P_i \) and \( Q_i \) are nonlinear functions of the state variables \( i \) and \( |V| \), power-flow calculations employs iterative techniques.

### The Newton-Rhapson Power Flow Solution

In this study the Newton Rhapson method (which is included as a tool within the PSS/E software [3] that is used during the simulation) is arbitrarily chosen to compute the power flow of the system, whose algorithm can be listed as the following [2]:

1. Make an initial guess for the unknown voltages: \( |V|^{(0)}, \delta^{(0)} \)
2. Compute the loadflow equations as eq. (4) and eq. (5) to obtain \( P_{\text{calc}}^{(0)} \) and \( Q_{\text{calc}}^{(0)} \)
3. Compute the power mismatches as
   \[
   \Delta P_i = P_{\text{specified}} - P_{\text{computed}} \quad (11)
   \]
   \[
   \Delta Q_i = Q_{\text{specified}} - Q_{\text{computed}} \quad (12)
   \]
4. Compute the Jacobian terms of
   \[
   J = \begin{bmatrix}
   \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_i}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial V_{1n}} & \cdots & \frac{\partial P_i}{\partial V_{1n}} \\
   \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
   \frac{\partial P_1}{\partial \delta_i} & \cdots & \frac{\partial P_i}{\partial \delta_i} & \cdots & \frac{\partial P_1}{\partial V_{in}} & \cdots & \frac{\partial P_i}{\partial V_{in}} \\
   \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
   \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_i}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial V_{1n}} & \cdots & \frac{\partial Q_i}{\partial V_{1n}} \\
   \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
   \frac{\partial Q_1}{\partial \delta_i} & \cdots & \frac{\partial Q_i}{\partial \delta_i} & \cdots & \frac{\partial Q_1}{\partial V_{in}} & \cdots & \frac{\partial Q_i}{\partial V_{in}} \\
   \end{bmatrix} \quad (13)
   \]
   Note that \( \Delta P_i \) and \( \Delta Q_i \) of the slack bus is undefined when \( P_i \) and \( Q_i \) are not schedule, since the slack bus serves as references for the angles of all other bus voltages, thus all terms involving \( \delta \) and \( \Delta |V| \) are omitted from the equations because those corrections are both zero at the slack bus.
5. Solve the corrections from the following system:
   \[
   \begin{bmatrix}
   \Delta \delta_1 \\
   \vdots \\
   \Delta \delta_i \\
   \Delta |V|_1 \\
   \vdots \\
   \Delta |V|_i \\
   \end{bmatrix} =
   J^{-1}
   \begin{bmatrix}
   \Delta P_1 \\
   \vdots \\
   \Delta P_i \\
   \Delta Q_1 \\
   \vdots \\
   \Delta Q_i \\
   \end{bmatrix}
   \quad (14)
   \]
6. Add the corrections to the previous value:
   \[
   \delta^{(n+1)} = \delta^{(n)} + \Delta \delta \\
   \]
   \[
   |V|^{(n+1)} = |V|^{(n)} + \Delta |V| \quad (15)
   \]
7. Use the new values \( \delta^{(i)} \) and \( |V|^{(i)} \) as starting values for iteration 2 and continue until the absolute values of the corrections smaller than a predefined value \( \varepsilon \).

### Stochastic DES

In the traditional 'vertically-operated' power system there are only a 'small' number of large centralized generators that are 'dispatchable'; controllable to meet the (predicted) demand. In the future 'horizontally-operated' power system, the DG units in the 'active' distribution networks, (called Distributed Energy Systems (DES) in this paper; see fig. 2: 'Loads and Distributed Generation'), are basically 'non-dispatchable'. This non-dispatchable behavior results from the fact that certain DG units generate power from primary energy sources with inherently stochastic characteristics, such as wind- and solar energy. But even when DG units are in essence 'deterministic', they can be customer-owned, and the owners can decide whether the units are running or not. In both cases the DG possesses a stochastic generation characteristic, and the modeling of the system needs to be adjusted to incorporate the stochastic behavior of DES for accurate load flow calculations. In this paper, DES with customer-owned generators are considered. The power flow solution of a system with this stochastic DG is computed by including the stochastic behavior of the parameters of the specified active \( P_{\text{sch}} \) and reactive \( Q_{\text{sch}} \) power of each bus \( i \) representing DES in a power system. With DG is implemented in every load bus and modelled as a negative load, in this structure, each load bus (i.e. distribution network) contains both consumption and generation (modelled as negative consumption) in the steady-state simulation. Therefore, seen from the transmission level, each distribution energy system (DES) can be represented as an aggregated load in parallel with aggregated generation, following:

\[
P_{\text{DES}(i)} = \sum_{j=1}^{L_i} P_{i(j)} - \sum_{k=1}^{N_{\text{DG}}} P_{DG(k,i)}, i = 1, \ldots, N_{\text{DES}} \quad (17)
\]

with \( N_{\text{DES}} \) is the number of DES, \( L_i \) is the number of loads in DES \( i \) and \( N_{\text{DG}} \) is the number of DG implemented in DES \( i \). The \( P_{\text{DES}(i)} \) value is then applied as \( P_{\text{sch}} \) in the Newton Rhapson algorithm described in earlier subsection, where considering that both the first and second terms of eq. 17 possess stochastic behavior, the \( P_{\text{DES}(i)} \) and the corresponding \( P_{\text{sch}} \) is also stochastic in its characteristic.
Research Approach

Test Systems

In this paper, a stochastic approach is applied in the load flow study of ‘horizontally-operated’ power systems with DG in order to investigate the impact of the stochastic behavior of DES on the network constraints (congestions). In other words: we try to find out whether the DES due to their stochastic behavior cause overloads in the power system. To illustrate the method, the simple 5-bus, 7-branch, test system (Hale network) [1] is used for the simulations. Fig. 3 shows the single line diagram of this system and table 1 specifies the scheduled (centralized) generation, the load, and the network parameters.

![Fig. 3 the 5-bus 7-branch test system [1]](image)

Table 1. Test system data [1]

<table>
<thead>
<tr>
<th>Node</th>
<th>PCG (MW)</th>
<th>QCG (Mvar)</th>
<th>PL (MW)</th>
<th>QL (Mvar)</th>
<th>Line</th>
<th>Zpq (10^-2 p.u.)</th>
<th>Ypq/2 (10^-2 p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 SLACK</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1-2</td>
<td>2+j6</td>
<td>3j</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>1-3</td>
<td>8+j24</td>
<td>2.5j</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>45</td>
<td>15</td>
<td>2-3</td>
<td>6+j18</td>
<td>2j</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>40</td>
<td>5</td>
<td>2-4</td>
<td>6+j18</td>
<td>2j</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>60</td>
<td>10</td>
<td>2-5</td>
<td>4+j32</td>
<td>1.5j</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3-4</td>
<td>1+j3</td>
<td>1j</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4-5</td>
<td>8+j24</td>
<td>2.5j</td>
</tr>
</tbody>
</table>

Simulation Scenario

To generate the samples that represent the values of the load of each DES (PL), using MCS, the following general assumptions are made:

- the consumption in each DES (PL) is modeled as a 2-state, high and low, load, where the high load is defined as the nominal power presented in table 1 and the low load state is considered to be 50% of the high load state
- each load level corresponds to a different time period of consumption (time-frame), with the low-load state covering 85% of the total time, regarded as a two-level time frames analysis [9], and
- each load at DES is following a normal distribution, where the mean values $\mu$ are the nominal 2-state load, and the standard deviations $\sigma$ equal 2% and 20% respectively for high and low loads.

Note that the MATLAB sub-routine command of $Rn = \text{normrnd}(MU,\text{SIGMA})$ that generates normal random numbers Rn with mean MU, i.e. $\mu$ which is equal to either one of the nominal 2-state load, standard deviation SIGMA, i.e. $\sigma$ which is equal to either 2% and 20% respectively for high and low loads, is basically used to generate the MCS random samples (10,000 samples are used in the MCS in this paper) of the active ($P_{Li}$) and reactive ($Q_{Li}$) power demands at every load bus $i$, and the MATLAB command of $Rb = \text{binornd}(Nb,pb)$ that generates random numbers from the binomial distribution with parameters specified by Nb, i.e. the maximal value of the random sample, Pb, i.e. $p$ which is the probability of the DG unit for being turned on, is basically used to generate the MCS random samples (10,000 samples) of the active power output ($P_{DG(i)}$) at every load bus $i$.

Fig. 4 shows the distribution of MCS generated samples (10,000 samples) representing the low and high loads at bus 2, 3, 4 and 5.

![Fig. 4. MCS generated samples representing the loads at each load bus](image)

Furthermore, to generate the samples representing the values of the aggregate DG at each DES ($P_{DG(i)}$), using MCS, some general assumptions are firstly defined as that:

- the DG units are customer-owned and randomly connected to and disconnected from the power system (based on the customers’ desires) [7], and
- the DG unit’s output power remains constant, at the nominal value, when connected to the power system.

Furthermore, more detail assumptions are defined as that:

- the total (aggregated) customer-owned DG is stochastically calculated as a binomial distribution.
where each DG unit within the DS is connected to the system with a probability $p$ that equals 0.8,

- the DG units supply only active power (1 MW nominal power each) and no reactive power, and
- the total nominal DG power in each DS equals the respective high load state, so that 45, 40, and 60 DG units are connected in DES 3, 4 and 5 respectively (see table 1).

Fig. 5 shows the distribution of MCS generated samples (10,000 samples) representing the aggregate DG power generation at bus 3, 4 and 5.

The flowchart of the complete computation is shown in fig. 6

**Simulation Results**

Fig. 7 shows the distribution of samples representing the DS as seen from the transmission system for the steady state analysis, when each of the MCS generated samples representing load and DG generation fig. 4 and fig. 5 are applied to eq. (17). It is evident from fig. 7, that in some cases each DS supplies positive active power to the transmission system. This occurs when the total aggregate power of the DG units exceeds the load.

![Fig. 5](image1.png)

**Fig. 5.** MCS generated samples representing the DG power generation at each DS (load bus)

The flowchart of the complete computation is shown in fig. 6

![Flowchart](image2.png)

**Fig. 6.** Computation to investigate the impact of stochastic DG on network constraints

**Simulation Results**

Fig. 7 shows the distribution of samples representing the DS as seen from the transmission system for the steady state analysis, when each of the MCS generated samples representing load and DG generation fig. 4 and fig. 5 are applied to eq. (17). It is evident from fig. 7, that in some cases each DS supplies positive active power to the transmission system. This occurs when the total aggregate power of the DG units exceeds the load.

![Fig. 7](image3.png)

**Fig. 7.** The net active power consumption of the DS

In fig. 8, the negative active (generated) power (i.e. consumed by the slack bus) means that more power is generated within the system than demanded by the loads (84.6% of the samples/cases). The positive active (generated) power (i.e. delivered by the slack bus) means that more power is demanded within the system than is generated by the DG and centralized generation (15.4% of the samples/cases). The traditional vertically operated power system is designed for uni-directional power flows. In the horizontally operated power system that we study in this paper the probability of having a bi-directional power flow is high (84.6%).

![Fig. 8](image4.png)

**Fig. 8.** The power delivery from the slack bus to the system

We can compare the maximum (29.8 MW) and minimum (-100.6 MW) power exchange between slack bus and system (see fig. 8) with the maximum and minimum values that result from a deterministic load flow calculation, which are respectively 11.5 MW (when all DG units are connected to test system and the load status is high) and -97 MW (when 100% of all DG units are connected to the test system and...
An Example of Utilizing Stochastic Approach for Investigating Network Constraints

M. Reza, et al.

Jurusan Teknik Elektro, Fakultas Teknologi Industri – Universitas Kristen Petra

http://www.petra.ac.id/~puslit/journals/dir.php?DepartmentID=ELK

85

the load status is low). It is evident that considering only the results from the deterministic load flow computation may lead to a misleading conclusion on the system performance.

Fig. 9 and fig. 10 display the statistical distributions of the active and complex power flows respectively in each line of the test system. As a comparison, in table 2 the active and complex power flows in each line in the base case of the test system (with no DG implemented) are given.

Fig. 9. The statistical distribution of the active power flow (in MW) in each line.

Fig. 10. The statistical distribution of the complex power flow (in MVA) in each line.

Table 2. Active and complex power flows in every line in the base test system (with no DG implemented).

<table>
<thead>
<tr>
<th>Line</th>
<th>High Load</th>
<th>Low Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>89.3</td>
<td>27.7</td>
</tr>
<tr>
<td>1-3</td>
<td>24.5</td>
<td>13.5</td>
</tr>
<tr>
<td>2-3</td>
<td>54.7</td>
<td>27.6</td>
</tr>
<tr>
<td>2-4</td>
<td>6.6</td>
<td>2.8</td>
</tr>
<tr>
<td>2-5</td>
<td>16.8</td>
<td>15.5</td>
</tr>
<tr>
<td>3-4</td>
<td>1.7</td>
<td>8.1</td>
</tr>
<tr>
<td>4-5</td>
<td>2.9</td>
<td>6.2</td>
</tr>
</tbody>
</table>

Fig. 5 MCS generated samples representing the DG power generation at each DS (load bus)

When we compare the active power flow on line 1-2 (between slack bus and system), the maximum base-case value (high load: 89.3 MW) is not exceeded in the case of the horizontally operated system; the distribution labeled 1-2 in fig. 9 is well below this value. However, if we compare the complex power flow on line 1-2 (between slack bus and system), the maximum base-case value (high load: 116 MW) is exceeded in almost 85% of the simulations in the case of the horizontally operated system (see the distribution labeled 1-2 in fig. 10). Therefore the results indicate that in the studied test system potential congestions can occur when the system is going to be operated ‘horizontally’.

Conclusions

In this paper an example of utilizing stochastic approach to study a problem within power system field is presented, where the case of investigating the impact of implementing distributed generation (DG) on the network constraints (congestions) is performed by including the stochastic behavior of the DG.

The results show that including the stochastic behavior of the DG give a more complete and detailed view of the possible congestions in the system than by using deterministic load flow computations only.

Acknowledgment

This research has been performed within the framework of the research program ‘intelligent power systems’ that is supported financially by Senter Novem. SenterNovem is an agency of the Dutch Ministry of Economic Affairs.

Reference

[8] M. Reza, P. H. Schavemaker, W. L. Kling, and L. Van der Sluis. A research program on intelligent power systems: Self controlling and