

# Power Flow in A Load-Current Sensorless Shunt Active Power Filter

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## ABSTRACT

In this paper, power flow analysis of a three-phase four-wire system with a shunt active power filter in steady state is presented. The analysis begins with a mathematical model of the power inverter and continues to find the relationship of the real and imaginary power as well as zero sequence power in the grid, loads, and the inverter (AC and DC sides) for successful compensation. The system includes mixed non-linear loads with significant unbalanced components. The filter consists of a three-phase current-controlled voltage source inverter (CC-VSI) with a filter inductance at the AC output and a DC-bus capacitor. The CC-VSI is operated to directly control the AC grid current to be sinusoidal and in phase with the grid voltage. Computer simulation results verify the concept of the filter and the power flow.

**Keywords:** active power filter, power flow

## ABSTRAK

*Dalam makala ini disajikan analisa aliran daya dari sistem tiga-fasa empat-kawat dengan sebuah shunt active power filter dalam keadaan ajek. Analisa dimulai dengan model matematika dari power inverter dan dilanjutkan dengan mencari hubungan antara daya nyata, imajiner, dan urutan nol pada sumber, beban, dan inverter (sisi AC dan DC) untuk proses kompensasi yang berhasil. Dalam sistem ini terdapat bermacam beban tidak linier yang juga tidak seimbang. Sedangkan filternya terdiri dari current-controlled voltage source inverter (CC-VSI) tiga fasa yang dilengkapi dengan induktansi filter pada keluaran AC dan kapasitor DC-bus. CC-VSI beroperasi untuk mengendalikan arus sumber AC secara langsung agar menjadi sinus dan sefasa dengan tegangan sumber. Hasil simulasi komputer bersesuaian dengan konsep dari filter dan aliran daya.*

**Kata kunci:** active power filter, aliran daya

## INTRODUCTION

Non-linear loads, especially power electronic loads, create harmonic currents and voltages in the power systems. For many years, various active power filters (APF) have been developed to suppress the harmonics, as well as compensate for reactive power, so that the utility grid will supply sinusoidal voltage and current with unity power factor [1][2].

In this paper, the three-phase shunt APF is a combination of a grid current-controlling shunt APF integrated with a series reactor installed at the Point of Common Coupling (PCC) to handle the harmonic and unbalance problems from mixed loads (see Figure 1) [3]. This novel shunt APF has been developed to overcome the problems occurred in the conventional shunt APF.

Conventionally, the power inverter as a shunt APF is controlled in such a way as to inject equal-but-opposite harmonic and reactive compensation currents based on calculated reference currents. Hence, the current sensors are installed on the load side. Then, their output signals will be processed to construct the reference or desired currents, which consist of harmonic and reactive components as well as negative- and zero-sequence components for unbalance compensation. Once the desired reference currents have been established, the currents must be injected into the grid accurately using a current control mechanism. The actual inverter currents must attempt to follow the reference currents with high bandwidth. In addition, the DC-bus voltage has to be kept constant to compensate for the inverter losses.

In contrast, the novel APF is operated to directly control the AC grid current to be sinusoidal and in phase with the grid voltage. By doing this, the three-phase shunt APF automatically provides compensation for harmonics, reactive (imaginary) power and unbalance.

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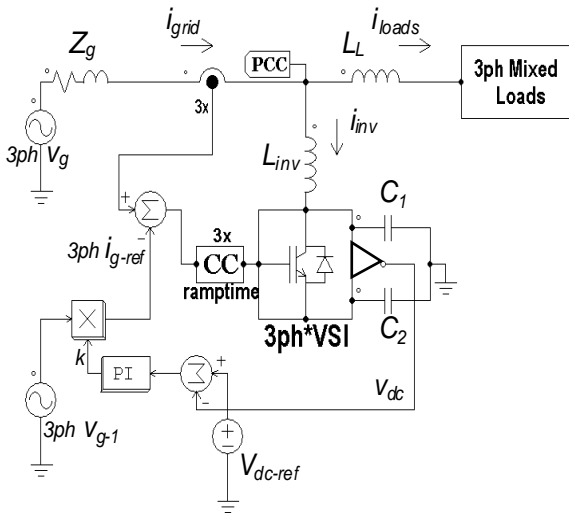


Figure 1. Block diagram of the shunt APF

There are many advantages of directly controlling the grid current. Firstly, it is easy to create a simple sinusoidal reference for the grid current. The reference current is an appropriate reference to minimize the grid harmonic currents. Secondly, the grid currents produced will be sinusoidal, balanced and in phase with the grid voltage regardless of grid voltage conditions. Thus, it prevents (more) pollution of the electrical system from non-linear loads. Furthermore, there are three current sensors installed at the grid side instead of six current sensors in a conventional shunt APF. The control mechanism becomes very simple as well.

Moreover, a shunt APF cannot properly compensate for harmonic voltage sources. In many cases, non-linear loads consist of combinations of harmonic voltage sources and harmonic current sources, and may contain significant load unbalance (ex. single phase loads on a three phase system). The addition of a small series inductance,  $X_L$  is simple, effective and practical not only to provide the required voltage decoupling between voltage type of load harmonic sources and the PCC but also to reduce the bandwidth of the load harmonic currents [4].

Due to its functions, the existence of the shunt APF may change the distribution of power between the grid and the load. Hence, power is flowing in the grid, the load and the inverter. It is expressed in terms of real power and imaginary power, which consist of an average value and an oscillating value. The relationship of these terms to the conventional concept about active, reactive and harmonic power can be seen in [5]. In this paper, power flow in the power system as well as in the shunt APF will be investigated.

## SHUNT ACTIVE POWER FILTER OPERATION

The three-phase shunt active power filter is a three-phase current-controlled voltage-source inverter (CC-VSI) with a mid-point earthed split capacitor ( $C_1$  and  $C_2$ ) in the DC bus and inductors ( $L_{inv}$ ) in the AC output. Thus, it is essentially three independent single-phase inverters with a common DC bus.

The APF consists of two control loops, namely an inner control loop and an outer control loop. The inner control loop is a ramp-time current control [6][7] that shapes the grid currents to be sinusoidal by generating a certain pattern of PWM for continuous switching of the inverter switches, while the outer control loop is a simple Proportional Integral (PI) control to keep the DC bus voltage constant and to provide the magnitude of reference current signals.

### Direct Control of the Grid Current

In this scheme (see Figure 1), the CC-VSI is operated to directly control the AC grid current rather than its own current. The grid current is sensed by putting the current sensors on the grid side and forced to behave as a sinusoidal current source. The grid appears as a high-impedance circuit for harmonics. By forcing the grid current to be sinusoidal, the APF automatically provides the harmonic, reactive, negative and zero sequence currents for the load, following the basic current summation rule:

$$i_{grid} = i_{inverter} + i_{loads} \quad (1)$$

The sinusoidal grid current reference signal is given by:

$$i_{g-ref} = k v_{grid-1} \quad (2)$$

where  $v_{grid-1}$  is the fundamental component of the grid voltage, and  $k$  is obtained from an outer control loop regulating the CC-VSI DC-bus voltage. This can be accomplished by a simple PI control loop. This is an effective way of determining the required magnitude of active current required, since any mismatch between the required load active current and that being forced by the CC-VSI would result in the necessary corrections to regulate the DC-bus voltage.

### Series Inductance [4]

Another key component of this system is the added series inductance  $X_L$ , which is comparable in value to the effective grid impedance,  $Z_g$ . Without this inductance, load harmonic voltage sources would produce harmonic currents through the grid impedance, which could not be compensated by a shunt

APF. Currents from the APF do not significantly change the harmonic voltage at the load. Therefore, there are still harmonic voltages across the grid impedance, which continue to produce harmonic currents. The inductance  $X_L$  provides the required voltage decoupling between load harmonic voltage sources and the grid.

### POWER INVERTER MODEL

The three-phase power distribution system with the shunt active power filter along with integrated series inductors is described in Figure 2. The devices are assumed to be the same for the three phases. Applying Kirchoff's rules to this system, the voltages of the grid, the load and the output inverter are given by:

$$v_{g-j} - v_{pcc-j} = R_g i_{g-j} + L_g \frac{di_{g-j}}{dt} \quad (3)$$

$$v_{pcc-j} - v_{L-j} = R_L i_{L-j} + L_L \frac{di_{L-j}}{dt} \quad (4)$$

$$v_{pcc-j} - v_{inv-j} = R_{inv} i_{inv-j} + L_{inv} \frac{di_{inv-j}}{dt} \quad (5)$$

For a three-phase system, the sum of the instantaneous three-phase voltages and currents is zero. The current relationship at the point of common coupling (PCC) is given by:

$$i_{g-j} = i_{inv-j} + i_{L-j} \quad (6)$$

where  $j = \{a, b, c\}$

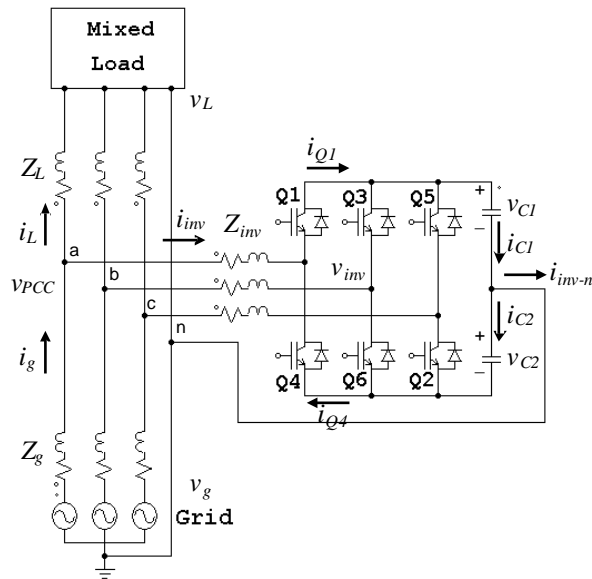


Figure 2. Power Circuit

By controlling the switches, the PWM CC-VSI output voltages  $v_{inv-j}$  can be expressed in terms of DC-

bus voltages (assuming the voltages are constant over the switching period) and the continuous duty ratios of the switches, which is denoted by  $d_j$ :

$$v_{inv-j} = d_j v_{C1} + (d_j - 1)v_{C2} \quad (7)$$

$$v_{dc} = v_{C1} + v_{C2} \quad (8)$$

The value of  $d_j$  varies from zero to one. Another expression for  $d_j$  in terms of the voltages can be found directly from equations (7) and (8):

$$d_j = \frac{v_{inv-j} + v_{C2}}{v_{dc}} \quad (9)$$

Therefore, as seen from the AC side, the power inverter (converter) can be modelled in time average as a voltage-controlled voltage source (VCVS) with the parameter of  $v_{inv}$ .

On the DC side, the modelling begins with the expression of the current through each bidirectional switch  $Q_j$ . For phase A, the average current  $i_{Q1}$  is equal to  $d_a i_{inv-a}$ , while  $i_{Q4}$  is equal to  $(d_a - 1) i_{inv-a}$ . Thus, in general, the average bidirectional switch currents can be written as functions of inverter output currents, namely:

$$i_{Q(top)-j} = d_j i_{inv-j} \quad (10)$$

$$i_{Q(bottom)-j} = (d_j - 1) i_{inv-j} \quad (11)$$

On the DC side, the current flowing to capacitor  $C_1$  is  $i_{C1}$ , and to capacitor  $C_2$  is  $i_{C2}$ . The currents  $i_{C1}$  and  $i_{C2}$  each are equal to the total currents through their respective bidirectional switches, and are simply expressed as follows:

$$i_{C1} = \sum_{j=a,b,c} d_j i_{inv-j} \quad (12)$$

$$i_{C2} = \sum_{j=a,b,c} (d_j - 1) i_{inv-j} \quad (13)$$

Thus, on the DC side, the converter is modelled in time average by a current-controlled current-source (CCCS) with DC-capacitor currents.

Equations (12) and (13) regarding DC-capacitor currents can be rearranged using Equations (5) and (9) to observe the influence of the parameters from the AC side of the inverter. After some algebraic manipulation, the currents in DC capacitors  $C_1$  and  $C_2$  are given by:

$$i_{C1} = \sum_{j=a,b,c} \frac{1}{v_{dc}} (v_{C2} i_{inv-j} + v_{pcc-j} i_{inv-j} - \quad (14)$$

$$L_{inv} \frac{di_{inv-j}}{dt} i_{inv-j} - R_{inv} i_{inv-j}^2)$$

$$i_{C2} = \sum_{j=a,b,c} \frac{1}{v_{dc}} (-v_{C1} i_{inv-j} + v_{pcc-j} i_{inv-j} - L_{inv} \frac{di_{inv-j}}{dt} i_{inv-j} - R_{inv} i_{inv-j}^2) \quad (15)$$

Based on equation (7) and equations (12) and (13), the inverter equivalent circuits can be illustrated as in Figure 3.

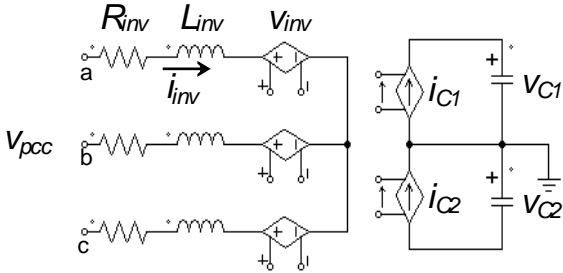


Figure 3. Equivalent Circuit of the Inverter

### POWER FLOW

Power flow analysis of this system in steady state is similar to that undertaken by Akagi [8][9]. Considering a non-sinusoidal and unbalanced system, the load power consists of real power  $p_L$ , imaginary power  $q_L$  and zero sequence power  $p_{L-0}$ , which consist of an average value and an oscillating value as mentioned above:

$$\begin{aligned} p_L &= \bar{p}_L + \tilde{p}_L \\ q_L &= \bar{q}_L + \tilde{q}_L \\ p_{L-0} &= \bar{p}_{L-0} + \tilde{p}_{L-0} \end{aligned} \quad (16)$$

The shunt active power filter has to compensate for the unwanted currents of the load so that after successful compensation, the grid currents will be sinusoidal and in-phase with the grid voltage. As a result, if the grid voltage is non-sinusoidal and/or unbalanced, the powers  $p_g$ ,  $q_g$  and  $p_{g-0}$  generated from the grid become:

$$\begin{aligned} p_g &= \bar{p}_g + \tilde{p}_g \\ q_g &= \tilde{q}_g \\ p_{g-0} &= 0 \end{aligned} \quad (17)$$

To achieve sinusoidal grid currents, both powers  $\tilde{p}_g$  and  $\tilde{q}_g$  still exist in the grid in small values. The power developed by the inverter is calculated by subtracting the power supplied by the grid and the power consumed by the load, given by:

$$\begin{bmatrix} p_{inv} \\ q_{inv} \\ p_{inv-0} \end{bmatrix} = \begin{bmatrix} p_g \\ q_g \\ p_{g-0} \end{bmatrix} - \begin{bmatrix} p_L \\ q_L \\ p_{L-0} \end{bmatrix} \quad (18)$$

From Equations (16) and (17), Equation (18) can be expressed in terms of average and oscillating values as:

$$\begin{bmatrix} p_{inv} \\ q_{inv} \\ p_{inv-0} \end{bmatrix} = \begin{bmatrix} \bar{p}_g - \bar{p}_L \\ -\bar{q}_L \\ -\bar{p}_{L-0} \end{bmatrix} + \begin{bmatrix} \tilde{p}_g - \tilde{p}_L \\ \tilde{q}_g - \tilde{q}_L \\ -\tilde{p}_{L-0} \end{bmatrix} \quad (19)$$

From Equation (19), the characteristics of the power flow in the system can be described as follows:

1. The inverter supplies the zero sequence average (active)  $\bar{p}_{L-0}$  and oscillating  $\tilde{p}_{L-0}$  power needed by the load.
2. To supply the load zero sequence active power, the inverter has to take an active power from the grid because the inverter has no DC source (only DC capacitors in the DC bus). Neglecting the losses in the power converter, in steady state, the active power consumed by the load is equal to the active power supplied by the grid, and total active power flowing to the inverter is zero. The active (average) power in Equation (19) can be combined as:  

$$\bar{p}_{inv} + \bar{p}_{inv-0} = \bar{p}_g - \bar{p}_L - \bar{p}_{L-0} = 0.$$
Because  $\bar{p}_{inv-0} = -\bar{p}_{L-0}$ , thus, the active power taken from the grid by the inverter, which is used to support the zero sequence power delivered to the load, is  $\bar{p}_{inv} = \bar{p}_g - \bar{p}_L = \bar{p}_{L-0}$ . Additional power consumption is required to compensate for the losses, so that  $\bar{p}_{inv} = \bar{p}_{L-0} + \bar{p}_{loss}$ .
3. The active power filter supplies  $\tilde{p}_L$  as well as  $\tilde{p}_{L-0}$  and consumes  $\tilde{p}_g$  using DC capacitors as an energy storage element. Hence, this power will appear in the DC-bus voltage ripple. Normally,  $\tilde{p}_g$  is small so that the DC-bus voltage predominantly reflects the load power.
4. The inverter controls the whole imaginary power associated with the load  $\bar{q}_L + \tilde{q}_L$  and the grid  $\tilde{q}_g$ .

### SIMULATION RESULTS

The system in Figure 1 is tested using computer simulation (PSIM) to verify the concept above. In this case, the grid voltage is made unbalanced. The fundamental component of the phase-A voltage is increased by 10%. The three-phase grid voltage is shown in Figure 4. The three-phase current waveforms of the mixed loads, as well as the neutral current from the computer simulation, are shown in Figure 5. Figure 6 demonstrates the steady-state performance of compensation results. It can be seen that the shunt APF is successfully able to compensate for the total mixed loads that produce harmonic,

reactive and unbalanced currents. It is obvious that the CC-VSI is able to generate three different currents for each phase as well as the neutral current, as shown in Figure 7. Hence, the inverter also provides balancing to create the symmetrical grid currents.

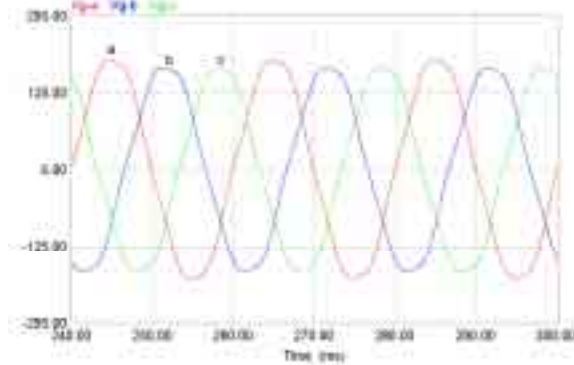


Figure 4. Grid voltages - unbalance and harmonics

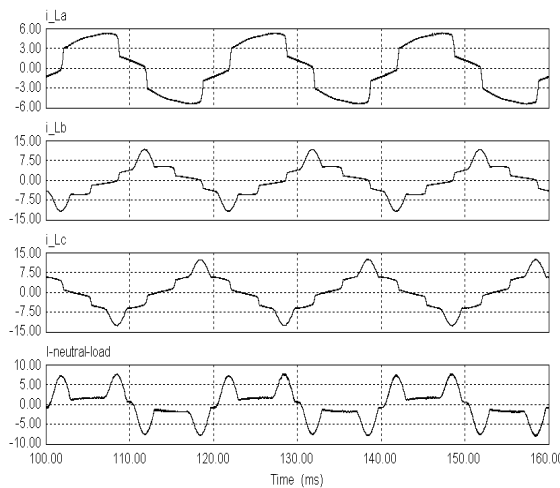


Figure 5. Load currents

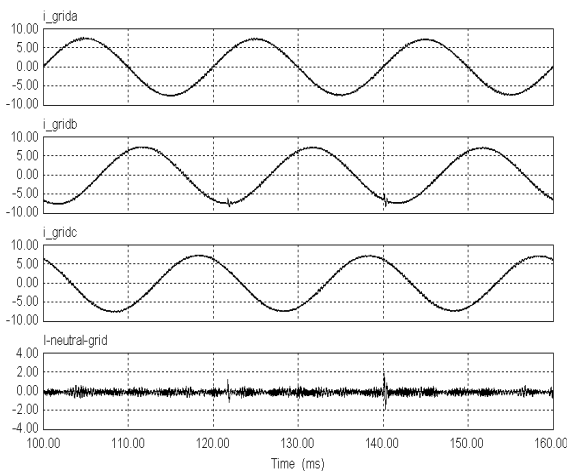


Figure 6. Grid currents after compensation

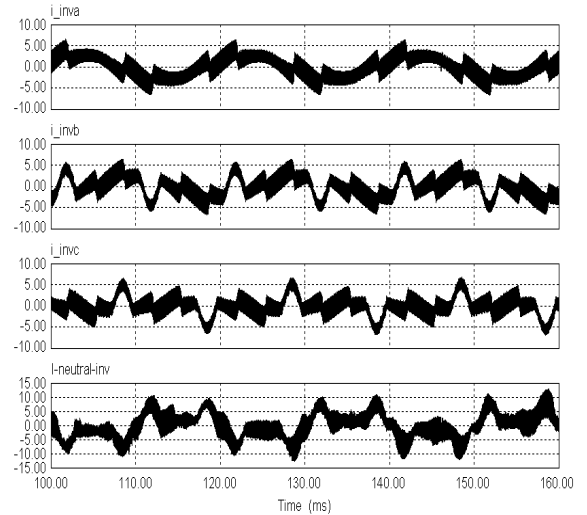


Figure 7. Inverter currents

Figure 8 shows the relationship of the real power from the grid, the load and the inverter. It is clear that  $p_{inv} = p_g - p_L$ . Both waveforms are the same and contain a small average value, which corresponds to zero sequence power as well as losses absorbed by the CC-VSI. Figure 9 shows that the zero sequence power of the load is compensated for by the CC-VSI, while the zero sequence power of the grid is close to zero. Thus, the grid supplies the required zero sequence active power to the inverter, and then the inverter delivers the power to the load. From Figure 10, the relationship of the imaginary power from the grid, the load and the inverter is demonstrated. Finally, the grid supplies the real power as shown in Figure 11, which agrees with Equation (17). The grid generates a small reactive power  $\bar{q}_g$  because the grid currents lead the voltages by approximately  $1^\circ$  due to the high-frequency switching filter. Shifting the grid reference current by a small negative value can eliminate the reactive power. Hence, computer simulation results verify the power flow in Equation (19).

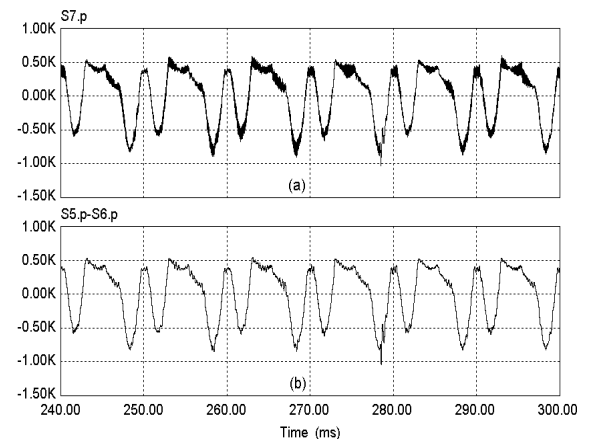


Figure 8. Real power (a)  $p_{inv}$  (b)  $p_g - p_L$

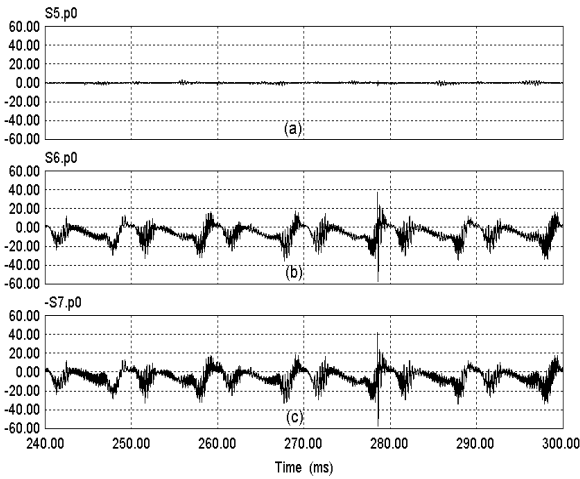


Figure 9. Zero sequence power (a)  $p_{g0}$  (b)  $p_{L0}$  (c)  $p_{inv0}$

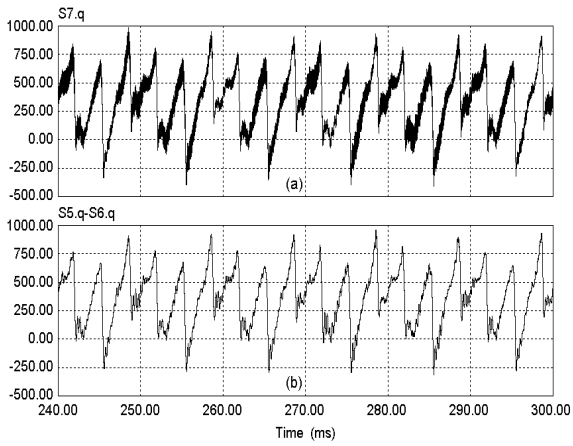


Figure 10. Imaginary power (a)  $q_{inv}$  (b)  $q_g - q_L$

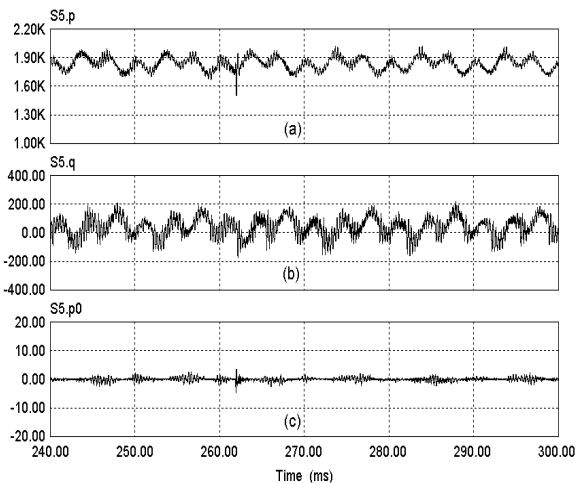


Figure 11. Power from grid (a) real power (b) imaginary power (c) zero sequence power

The power characteristics of the inverter have to be reflected in the power at the DC bus. The power at the DC bus can be given by:

$$P_{dc-bus} = v_{C1} i_{C1} + v_{C2} i_{C2} \quad (20)$$

Where  $i_{C1}$  and  $i_{C2}$  are taken from Equations (14) and (15):

$$v_{C1} i_{C1} = v_{C1} \sum_{j=a,b,c} \frac{1}{v_{dc}} (v_{C2} i_{inv-j} + v_{pcc-j} i_{inv-j} - L_{inv} \frac{di_{inv-j}}{dt} i_{inv-j} - R_{inv} i_{inv-j}^2) \quad (21)$$

$$v_{C2} i_{C2} = v_{C2} \sum_{j=a,b,c} \frac{1}{v_{dc}} (-v_{C1} i_{inv-j} + v_{pcc-j} i_{inv-j} - L_{inv} \frac{di_{inv-j}}{dt} i_{inv-j} - R_{inv} i_{inv-j}^2) \quad (22)$$

Substituting Equations (21) and (22) into (20) for  $P_{dc-bus}$  yields:

$$P_{dc-bus} = \frac{(v_{C1} + v_{C2})}{v_{dc}} \left[ \sum_{j=a,b,c} v_{pcc-j} i_{inv-j} - L_{inv} \frac{di_{inv-j}}{dt} i_{inv-j} - R_{inv} i_{inv-j}^2 \right] \quad (23)$$

The DC capacitor currents may contain a zero sequence current; however, the power at the DC-bus is free from zero sequence current. From (23), the power at the DC bus predominantly depends upon the first part of the equation. Multiplying  $v_{pcc}$  and  $i_{inv}$  at the same phase results in instantaneous real power and instantaneous zero sequence power. The zero sequence power can exist in the DC bus depending upon both the zero sequence current from the load and the zero sequence voltage at the PCC.

The DC bus does not contain imaginary power. This fact agrees with the concept of instantaneous reactive power compensation using switching devices without energy storage [7]. The CC-VSI generates  $q_{inv}$ , but it does not flow out of or into the DC-bus capacitors. According to Watanabe [9] and Peng [10], the imaginary power circulates among the phases. In other words, instantaneously, the imaginary power required by one phase can be supplied by the other phase.

The DC-bus capacitors absorb the small active power to compensate for the losses in order to keep the DC voltage constant.

## CONCLUSION

This paper explained the operation and the mathematical model of the CC-VSI as a three-phase shunt active power filter. The power inverter is operated to directly control the grid currents to be sinusoidal and in phase with the grid voltage. For the

model, on the AC side, the CC-VSI can be represented as a voltage-controlled voltage source; while on the DC side, a current-controlled current source represents the inverter.

From power flow analysis, the power distribution in the grid, loads and the inverter is identical to the system with conventional APF. It is obvious that the shunt APF is able to enhance the power capacity of the grid by compensating for the non-active power. The inverter controls the imaginary power and the zero sequence power. However, the non-active power does not emerge in the DC bus. The DC bus only contains real power as well as zero sequence power. Computer simulation and experimental results clarify the theoretical observations.

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